Charles’s Law
• At constant pressure \((P)\) the volume \((V)\) of a fixed amount of gas is proportional to its absolute temperature \((T)\)
  \[\Rightarrow \text{At constant } P \text{ and } n:\]
  \[V = k'T \quad k' \rightarrow \text{constant (depends on } P \text{ and } n)\]
  \[\frac{V}{T} = k' \quad T \uparrow \Leftrightarrow V \uparrow\]
• Charles’s law helped devise the absolute temperature scale (Lord Kelvin)

• Assume two states of a gas at constant \(P\)
  – state 1 \(\rightarrow\) \(T_1, V_1\)
  – state 2 \(\rightarrow\) \(T_2, V_2\)

\[
\frac{V_1}{T_1} = k' \quad \frac{V_2}{T_2} = k' \quad \Rightarrow \quad \frac{V_1}{T_1} = \frac{V_2}{T_2}
\]

Example: A balloon is filled with 5.0 L He gas at 15°C. The temperature is changed to 35°C. What is the new volume of the balloon?

\(T_1 = 15°C = 288 \text{ K}\) \(V_1 = 5.0 \text{ L}\)
\(T_2 = 35°C = 308 \text{ K}\) \(V_2 = ?\)

\[
V_2 = \frac{V_1T_2}{T_1} = \frac{5.0 \text{ L} \times 308 \text{ K}}{288 \text{ K}} = 5.3 \text{ L}
\]

Variations of Charles’s law – Amontons’s law
• At constant volume \((V)\) the pressure \((P)\) of a fixed amount of gas is proportional to its absolute temperature \((T)\)
  \[\Rightarrow \text{At constant } V \text{ and } n:\]
  \[P = k''T \quad k'' \rightarrow \text{constant (depends on } V \text{ and } n)\]
  \[\frac{P}{T} = k'' \quad T \uparrow \Leftrightarrow P \uparrow\]
• Assume two states of a gas at constant $V$
  - state 1 $\rightarrow T_1, P_1$
  - state 2 $\rightarrow T_2, P_2$

$$\frac{P_1}{T_1} = k'' \quad \frac{P_2}{T_2} = k'' \quad \Rightarrow \quad \frac{P_1}{T_1} = \frac{P_2}{T_2}$$

**Example:** A cylinder containing $N_2$ gas at 15°C and 50 atm is moved to a new location at 35°C. What is the new pressure in the cylinder?

$T_1 = 15^\circ C = 288 \text{ K}$

$P_1 = 50 \text{ atm}$

$T_2 = 35^\circ C = 308 \text{ K}$

$P_2 = ?$

$$P_2 = \frac{P_1 T_2}{T_1} = \frac{50 \text{ atm} \times 308 \text{ K}}{288 \text{ K}} = 53 \text{ atm}$$

• **Avogadro’s Law**
  - At constant temperature ($T$) and pressure ($P$) the volume ($V$) of a gas is proportional to its amount ($n$)
  - At constant $T$ and $P$:

$$V = k'''n \quad k''' \rightarrow \text{constant (depends on } T, P)$$

$$V \Rightarrow n \leftrightarrow V$$

• Molar volume ($V_m$) – the volume of one mole of a substance

$$V_m = \frac{V}{n}$$

• **Avogadro’s principle**
  - At constant $T$ and $P$ equal number of moles of different gases occupy equal volumes
  - Molar volumes of gases are very similar ($V/n = \text{constant}$)

• Assume two states of a gas at constant $T$ and $P$
  - state 1 $\rightarrow V_1, n_1$
  - state 2 $\rightarrow V_2, n_2$

$$\frac{V_1}{n_1} = k''' \quad \frac{V_2}{n_2} = k''' \quad \Rightarrow \quad \frac{V_1}{n_1} = \frac{V_2}{n_2}$$

**The Ideal Gas Law**

- **Boyle's Law**
  $$V = k \frac{1}{P}$$

- **Charles's Law**
  $$V = k'T$$

- **Avogadro's Law**
  $$V = k'''n$$

• Combination of the three laws:

$$V = R \frac{nT}{P} \quad R \rightarrow \text{proportionality constant}$$

$$PV = nRT \rightarrow \text{ideal gas law}$$

$R \rightarrow \text{universal gas constant}$
There is no need to memorize the mathematical expression for the individual gas laws since they can all be derived from the ideal gas law.

- **Ideal gas** – obeys the ideal gas law
- **R** is determined experimentally
  
  \[ R = 0.08206 \text{ L·atm/mol·K} \]
  \[ R = 8.314 \text{ J/mol·K} \]

- Assume two states of a gas
  - state 1 \( P_1, V_1, n_1, T_1 \)
  - state 2 \( P_2, V_2, n_2, T_2 \)

\[
\frac{P_1V_1}{n_1T_1} = R \quad \frac{P_2V_2}{n_2T_2} = R \quad \Rightarrow \quad \frac{P_1V_1}{n_1T_1} = \frac{P_2V_2}{n_2T_2}
\]

Note: \( T \) must always be in Kelvin

**Example:** A 5.0 L gas sample at 1.0 atm and 10ºC is moved to a 2.0 L container and heated to 300ºC. What is the new pressure?

\( P_1 = 1.0 \text{ atm} \quad V_1 = 5.0 \text{ L} \quad T_1 = 10\text{ºC} = 283 \text{ K} \)

\( P_2 = \text{?} \quad V_2 = 2.0 \text{ L} \quad T_2 = 300\text{ºC} = 573 \text{ K} \)

\[
n_1 = n_2
\]

\[
\frac{P_1V_1}{n_1T_1} = \frac{P_2V_2}{n_2T_2} \quad \Rightarrow \quad P_2 = \frac{P_1V_1}{n_1T_1} \times \frac{n_2T_2}{V_2}
\]

\[
P_2 = \frac{P_1V_1T_2}{T_1V_2} = \frac{1.0 \text{ atm} \times 5.0 \text{ L} \times 573 \text{ K}}{283 \text{ K} \times 2.0 \text{ L}} = 5.1 \text{ atm}
\]

**Example:** A 3.0 g sample of methane, \( \text{CH}_4 \), is placed in a 2.0 L container at 22ºC. What is the pressure in the container?

\[ PV = nRT \]

\( V = 2.0 \text{ L} \quad T = 22\text{ºC} = 295 \text{ K} \)

moles of \( \text{CH}_4 \) (\( n \)):

\[
n = 3.0 \text{ g CH}_4 \times \frac{1 \text{ mol CH}_4}{16.0 \text{ g CH}_4} = 0.19 \text{ mol CH}_4
\]

\[
P = \frac{nRT}{V} = \frac{0.19 \text{ mol} \times 0.08206 \frac{\text{L·atm}}{\text{mol·K}} \times 295 \text{ K}}{2.0 \text{ L}} = 2.3 \text{ atm}
\]
Standard conditions

• Standard temperature and pressure (STP)
  \( P = 1 \text{ atm}; \ T = 0^{\circ}\text{C} = 273.15 \text{ K} \)

• The molar volume of the ideal gas at STP

\[
V_m = \frac{V}{n} = \frac{nRT}{P} = \frac{RT}{P}
\]

\[
V_m = \frac{RT}{P} = \frac{0.08206 \text{ L} \cdot \text{atm}}{\text{mol} \cdot \text{K}} \times 273.15 \text{ K} = 22.41 \text{ L/mol}
\]