Charles's Law

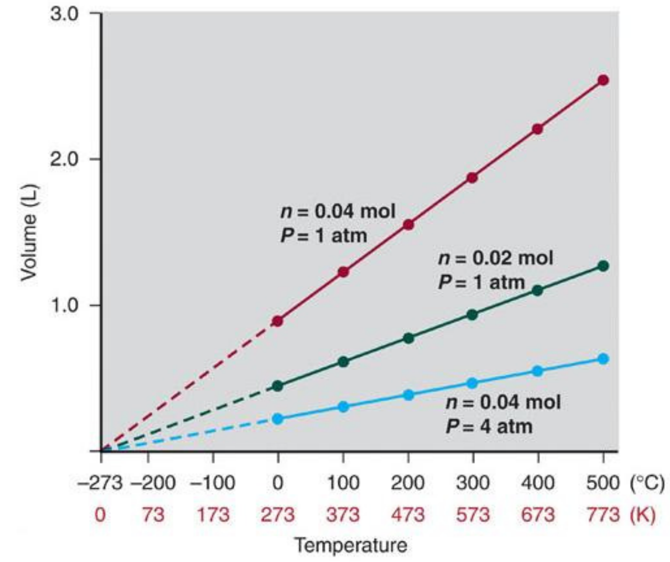
- At constant pressure (*P*) the volume (*V*) of a fixed amount of gas is proportional to its absolute temperature (*T*)

  ⇒ At constant *P* and *n*:

$V = k'T$ **$k' \rightarrow$ constant (depends on *P* and *n*)**

$$\frac{V}{T} = k' \qquad T \uparrow \Leftrightarrow V \uparrow$$

- Charles's law helped devise the absolute temperature scale (Lord Kelvin)



- Assume two states of a gas at constant ***P***
  - state 1 → T_1, V_1
  - state 2 → T_2, V_2

$$\frac{V_1}{T_1} = k' \qquad \frac{V_2}{T_2} = k' \qquad \Rightarrow \qquad \frac{V_1}{T_1} = \frac{V_2}{T_2}$$

Example: A balloon is filled with 5.0 L He gas at 15ºC. The temperature is changed to 35ºC. What is the new volume of the balloon?

T_1 = 15ºC = 288 K **V_1 = 5.0 L**

T_2 = 35ºC = 308 K **V_2 = ?**

$$V_2 = \frac{V_1 T_2}{T_1} = \frac{5.0\ \text{L} \times 308\ \text{K}}{288\ \text{K}} = 5.3\ \text{L}$$

Variations of Charles's law – Amontons's law

- At constant volume (*V*) the pressure (*P*) of a fixed amount of gas is proportional to its absolute temperature (*T*)

  ⇒At constant *V* and *n*:

$P = k''T$

$k'' \rightarrow$ constant (depends on *V* and *n*)

$$\frac{P}{T} = k'' \qquad T \uparrow \Leftrightarrow P \uparrow$$

- Assume two states of a gas at constant V
  - state 1 → T_1, P_1
  - state 2 → T_2, P_2

$$\frac{P_1}{T_1} = k'' \qquad \frac{P_2}{T_2} = k'' \qquad \Rightarrow \qquad \frac{P_1}{T_1} = \frac{P_2}{T_2}$$

Example: A cylinder containing N_2 gas at 15ºC and 50 atm is moved to a new location at 35ºC. What is the new pressure in the cylinder?

T_1 = 15ºC = 288 K $\qquad$ P_1 = 50 atm

T_2 = 35ºC = 308 K $\qquad$ P_2 = ?

$$P_2 = \frac{P_1 T_2}{T_1} = \frac{50\ \text{atm} \times 308\ \text{K}}{288\ \text{K}} = 53\ \text{atm}$$

Avogadro's Law

- At constant temperature (T) and pressure (P) the volume (V) of a gas is proportional to its amount (n)
  - At constant T and P:

$V = k'''n$ $\qquad$ k''' → **constant (depends on T, P)**

$$\frac{V}{n} = k''' \qquad n\uparrow \Leftrightarrow V\uparrow$$

- Molar volume (V_m) – the volume of one mole of a substance

$$V_m = V/n$$

- Avogadro's principle
  - At constant T and P equal number of moles of different gases occupy equal volumes
  - Molar volumes of gases are very similar (V/n = constant)
- Assume two states of a gas at constant T and P
  - state 1 → V_1, n_1
  - state 2 → V_2, n_2

$$\frac{V_1}{n_1} = k''' \qquad \frac{V_2}{n_2} = k''' \qquad \Rightarrow \qquad \frac{V_1}{n_1} = \frac{V_2}{n_2}$$

The Ideal Gas Law

$V = k\dfrac{1}{P}$ $\qquad$ **Boyle's Law**

$V = k'T$ $\qquad$ **Charles's Law**

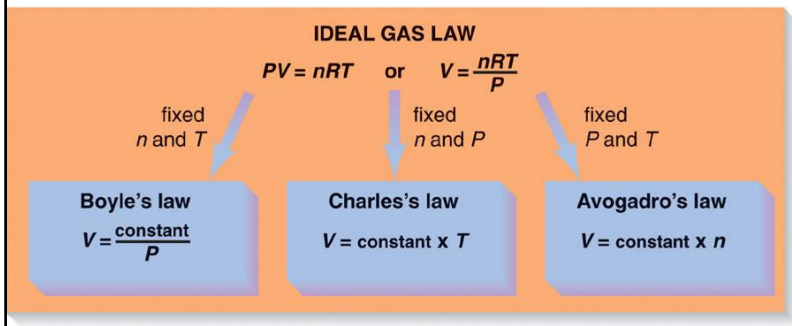
$V = k'''n$ $\qquad$ **Avogadro's Law**

- Combination of the three laws:

$V = R\dfrac{nT}{P}$ $\qquad$ R – **proportionality constant**

$PV = nRT$ → **ideal gas law**

R → **universal gas constant**



⇒ There is no need to memorize the mathematical expression for the individual gas laws since they can all be derived from the ideal gas law

- **Ideal gas** – obeys the ideal gas law
- ***R*** is determined experimentally
  - $R = 0.08206$ L·atm/mol·K
  - $R = 8.314$ J/mol·K
- Assume two states of a gas
  - state 1 → P_1, V_1, n_1, T_1
  - state 2 → P_2, V_2, n_2, T_2

$$\frac{P_1V_1}{n_1T_1} = R \qquad \frac{P_2V_2}{n_2T_2} = R \qquad \Rightarrow \qquad \frac{P_1V_1}{n_1T_1} = \frac{P_2V_2}{n_2T_2}$$

Note: *T* must always be in Kelvin

Example: A 5.0 L gas sample at 1.0 atm and 10ºC is moved to a 2.0 L container and heated to 300ºC. What is the new pressure?

$P_1 = 1.0$ atm $\qquad V_1 = 5.0$ L $\qquad T_1 = 10$ºC $= 283$ K

$P_2 = ?$ $\qquad V_2 = 2.0$ L $\qquad T_2 = 300$ºC $= 573$ K

$$n_1 = n_2$$

$$\frac{P_1V_1}{n_1T_1} = \frac{P_2V_2}{n_2T_2} \qquad \Rightarrow \qquad P_2 = \frac{P_1V_1}{\cancel{n_1}T_1} \times \frac{\cancel{n_2}T_2}{V_2}$$

$$P_2 = \frac{P_1V_1T_2}{T_1V_2} = \frac{1.0\text{ atm} \times 5.0\ \cancel{\text{L}} \times 573\ \cancel{\text{K}}}{283\ \cancel{\text{K}} \times 2.0\ \cancel{\text{L}}} = 5.1\text{ atm}$$

Example: A 3.0 g sample of methane, CH_4, is placed in a 2.0 L container at 22ºC. What is the pressure in the container?

$PV = nRT$

$V = 2.0$ L $\qquad T = 22$ºC $= 295$ K

moles of CH_4 (*n*):

$$n = 3.0\text{ g }CH_4 \times \left(\frac{1\text{ mol }CH_4}{16.0\text{ g }CH_4}\right) = 0.19\text{ mol }CH_4$$

$$P = \frac{nRT}{V} = \frac{0.19\ \cancel{\text{mol}} \times 0.08206\ \frac{\cancel{\text{L}}\cdot\text{atm}}{\cancel{\text{mol}}\cdot\cancel{\text{K}}} \times 295\ \cancel{\text{K}}}{2.0\ \cancel{\text{L}}} = 2.3\text{ atm}$$

Standard conditions

- Standard temperature and pressure (STP)

  P = 1 atm; T = 0ºC = 273.15 K

- The molar volume of the ideal gas at STP

$$V_m = \frac{V}{n} = \frac{nRT/P}{n} = \frac{RT}{P}$$

$$V_m = \frac{RT}{P} = \frac{0.08206\ \frac{\text{L}\cdot\text{atm}}{\text{mol}\cdot\text{K}} \times 273.15\ \text{K}}{1\ \text{atm}} = 22.41\ \frac{\text{L}}{\text{mol}}$$