5.4 Applications of the Ideal Gas Law

The molar mass and density of gasses

$$\text{density} = \frac{\text{mass}}{\text{volume}} \quad \rightarrow \quad d = \frac{m}{V}$$

$$\text{molar mass} = \frac{\text{mass}}{\text{moles}} \quad \rightarrow \quad M = \frac{m}{n} \quad \rightarrow \quad m = nM$$

$$PV = nRT \quad \rightarrow \quad n = \frac{PV}{RT}$$

$$d = \frac{m}{V} = \frac{nM}{V} = \frac{PVM}{RTV} \quad \rightarrow \quad d = \frac{MP}{RT} \quad \rightarrow \quad M = \frac{dRT}{P}$$

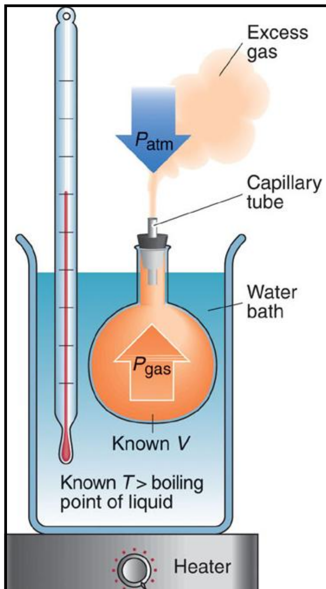
- The density of a gas is proportional to its molar mass and pressure and inversely proportional to its temperature

Example: Calculate the density of O_2 at STP.

$M = 32.00$ **g/mol**

$P = 1$ **atm** $\qquad T = 0^{o}\text{C} = 273.15$ **K** $\qquad$ **(STP)**

$$d = \frac{MP}{RT} = \frac{32.00\,\frac{\text{g}}{\text{mol}} \times 1\ \text{atm}}{0.08206\,\frac{\text{L}\cdot\text{atm}}{\text{mol}\cdot\text{K}} \times 273.15\ \text{K}} = 1.428\,\frac{\text{g}}{\text{L}}$$



- Finding the molar mass of a volatile liquid
  - Weigh a flask with a known volume
  - Fill the flask with the vapors of the volatile liquid at a known temperature and pressure
  - Cool the flask and let the vapors condense
  - Reweigh the flask to get the mass of the vapors

Example: Calculate the molar mass of a liquid if **0.955 g** of its vapors occupy **2.50 L** at **200ºC** and **45.0 Torr**.

$d = m/V = 0.955$ **g/2.50 L = 0.382 g/L**

$T = 200^{o}\text{C} = 473$ **K**

$P = 45.0$ **Torr×[1 atm/760 Torr] = 0.0592 atm**

$$M = \frac{dRT}{P} = \frac{0.382\,\frac{\text{g}}{\text{L}} \times 0.08206\,\frac{\text{L}\cdot\text{atm}}{\text{mol}\cdot\text{K}} \times 473\ \text{K}}{0.0592\ \text{atm}} = 250\,\frac{\text{g}}{\text{mol}}$$

Mixtures of Gasses

- Mixtures are treated just like pure gases – same gas laws apply
- **Partial pressure** of a gas in a mixture – the pressure the gas would exert if it occupied the container alone
- Dalton's **law of partial pressures** – the total pressure (***P***) of a gaseous mixture is the sum of the partial pressures (***P**$_i$*) of its components

$$P = P_A + P_B + \dots \quad \text{or} \quad \boxed{P = \sum P_i}$$

- **Mole fraction** (χ_i) of a gas in a mixture – a fraction of the total number of moles that belongs to that gas

$$\boxed{\chi_i = \frac{n_i}{\sum n_i} = \frac{n_i}{n}} \qquad \sum n_i = n \qquad \sum \chi_i = 1$$

- The sum of all mol fractions is equal to one
- The ideal gas law can be written for each gas in a mixture in terms of partial pressures

$$P_i V = n_i RT \qquad PV = nRT$$

$$\frac{P_i V}{PV} = \frac{n_i RT}{nRT} \qquad \frac{P_i}{P} = \frac{n_i}{n} = \chi_i$$

$$\boxed{P_i = \chi_i P}$$

⇒The partial pressure of a gas is proportional to its mol fraction

Example: Calculate the total pressure and the partial pressures of He and Ne in a **2.0 L** mixture containing **1.0 g He** and **2.0 g Ne** at **20°C**.

grams of He and Ne → moles of He and Ne → mole fractions of He and Ne

total pressure → partial pressures

$$1.0\ \text{g He} \times \left(\frac{1\ \text{mol He}}{4.00\ \text{g He}}\right) = 0.25\ \text{mol He}$$

$$2.0\ \text{g Ne} \times \left(\frac{1\ \text{mol Ne}}{20.18\ \text{g Ne}}\right) = 0.099\ \text{mol Ne}$$

$$\chi_{He} = \frac{n_{He}}{n_{He} + n_{Ne}} = \frac{0.25}{0.25 + 0.099} = 0.72$$

$$\chi_{Ne} = \frac{n_{Ne}}{n_{He} + n_{Ne}} = \frac{0.099}{0.25 + 0.099} = 0.28$$

$$n = 0.25 + 0.099 = 0.35 \text{ mol}$$

$$PV = nRT \quad \rightarrow \quad P = \frac{nRT}{V}$$

$$P = \frac{nRT}{V} =$$

$$= \frac{0.35 \text{ mol} \times 0.08206 \frac{\text{L} \cdot \text{atm}}{\text{mol} \cdot \text{K}} \times 293 \text{ K}}{2.0 \text{ L}} =$$

$$= 4.2 \text{ atm}$$

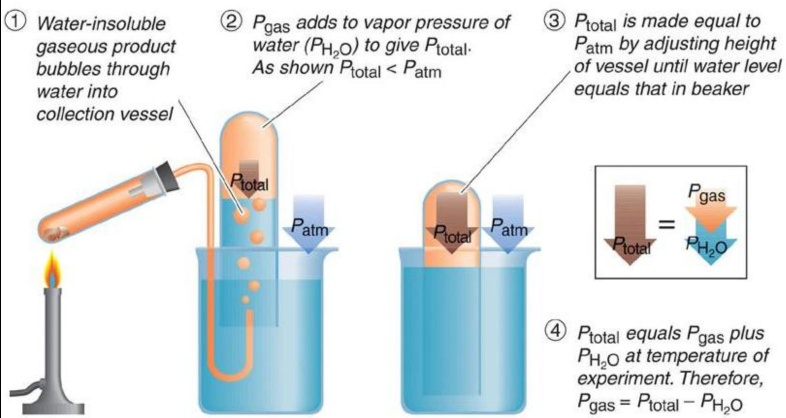
$$P_{He} = \chi_{He} P = 0.72 \times 4.2 \text{ atm} = 3.0 \text{ atm}$$

$$P_{Ne} = \chi_{Ne} P = 0.28 \times 4.2 \text{ atm} = 1.2 \text{ atm}$$

- Collecting a gas over water

$P_{\text{total}} = P_{\text{gas}} + P_{\text{water}}$ $\qquad$ $P_{\text{total}} = P_{\text{atm}}$

P_{water} **(vapor pressure of water) → given in tables**



Example: A **2.5 L** sample of O_2 gas was collected over water at **26°C** and **745 torr** atmospheric pressure. What is the mass of O_2 in the sample? (The vapor pressure of water at 26°C is **25 torr**.)

$P_{\text{oxygen}} = P_{\text{total}} - P_{\text{water}} = 745 - 25 = 720$ **torr**

$T = 26 + 273.15 = 299$ **K**

$$n_{O_2} = \frac{P_{O_2} V}{RT} = \frac{720 \cancel{\text{ torr}} \times \left(\frac{1 \cancel{\text{atm}}}{760 \cancel{\text{ torr}}} \right) \times 2.5 \cancel{\text{ L}}}{0.08206 \frac{\cancel{\text{L} \cdot \text{atm}}}{\text{mol} \cdot \cancel{\text{K}}} \times 299 \cancel{\text{ K}}} = 0.097 \text{ mol}$$

$$0.097 \text{ mol } O_2 \times \left(\frac{32.0 \text{ g } O_2}{1 \text{ mol } O_2} \right) = 3.1 \text{ g } O_2$$

Stoichiometry and the Ideal Gas Law

- The volume ratios of gases in reactions are the same as their mole ratios (follows from Avogadro's principle)

$$3H_2(g) + N_2(g) \rightarrow 2NH_3(g)$$

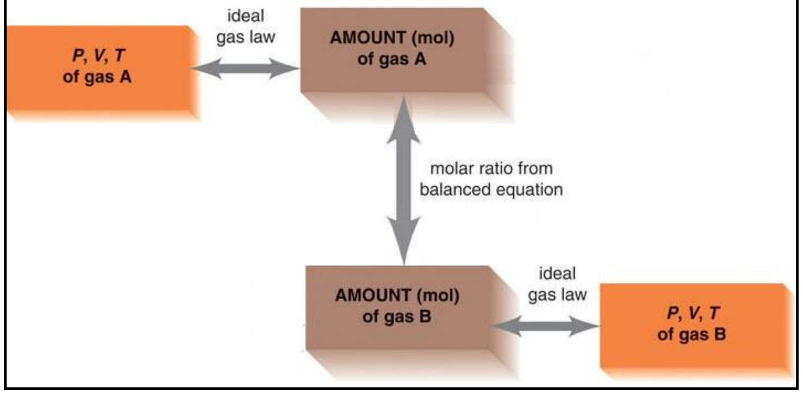
$\Rightarrow$ **3 mol H_2 react with 1 mol N_2**

$\Rightarrow$ **3 L H_2 react with 1 L N_2**

Example: How many liters of N_2 are needed to react completely with 5.0 L H_2?

$$5.0\ \text{L H}_2 \times [1\ \text{L N}_2 / 3\ \text{L H}_2] = 1.7\ \text{L N}_2$$

- The ideal gas law can be used to convert between the number of moles of gaseous reactants (or products) and their volumes at certain T and P



Example: Calculate the volume of CO_2 produced by the decomposition of **2.0 g** $CaCO_3$ at **25ºC** and **1.0 atm**.

$$CaCO_3(s) \rightarrow CaO(s) + CO_2(g)$$

$$2.0\ \text{g CaCO}_3 \times \left(\frac{1\ \text{mol CaCO}_3}{100.1\ \text{g CaCO}_3}\right) \times \left(\frac{1\ \text{mol CO}_2}{1\ \text{mol CaCO}_3}\right)$$

$$= 0.020\ \text{mol CO}_2$$

$$V = \frac{nRT}{P} = \frac{0.020\ \text{mol} \times 0.08206 \frac{\text{L}\cdot\text{atm}}{\text{mol}\cdot\text{K}} \times 298\ \text{K}}{1.0\ \text{atm}}$$

$$= 0.49\ \text{L}$$

Example: Calculate the mass of NaN_3 needed to produce 10 L of N_2 in an air bag at 25ºC and 1.0 atm by the reaction:

$$6NaN_3(s) + 2Fe_2O_3(s) \rightarrow 3Na_2O_2(s) + 4Fe(s) + 9N_2(g)$$

$T = 298$ K $\quad P = 1$ atm $\quad V = 10$ L $\quad n = ?$

$$n = \frac{PV}{RT} = \frac{1\ \text{atm} \times 10\ \text{L}}{0.08206 \frac{\text{L}\cdot\text{atm}}{\text{mol}\cdot\text{K}} \times 298\ \text{K}} = 0.41\ \text{mol}$$

$$0.41\ \text{mol N}_2 \left(\frac{6\ \text{mol NaN}_3}{9\ \text{mol N}_2}\right)\left(\frac{65.02\ \text{g NaN}_3}{1\ \text{mol NaN}_3}\right)$$

$$= 18\ \text{g NaN}_3$$