5.4 Applications of the Ideal Gas Law

The molar mass and density of gasses

\[
\text{density} = \frac{\text{mass}}{\text{volume}} \quad \Rightarrow \quad d = \frac{m}{V}
\]

\[
\text{molar mass} = \frac{\text{mass}}{\text{moles}} \quad \Rightarrow \quad M = \frac{m}{n} \quad \Rightarrow \quad m = nM
\]

\[
PV = nRT \quad \Rightarrow \quad n = \frac{PV}{RT}
\]

\[
d = \frac{m}{V} = \frac{nM}{V} = \frac{PVM}{RTV} \quad \Rightarrow \quad d = \frac{MP}{RT} \quad \Rightarrow \quad M = \frac{dRT}{P}
\]

- The density of a gas is proportional to its molar mass and pressure and inversely proportional to its temperature

**Example:** Calculate the density of O₂ at STP.

\[
M = 32.00 \text{ g/mol} \\
P = 1 \text{ atm} \\
T = 0^\circ \text{C} = 273.15 \text{ K} \quad \text{(STP)}
\]

\[
d = \frac{MP}{RT} = \frac{32.00 \text{ g/mol} \times 1 \text{ atm}}{0.08206 \frac{\text{L} \cdot \text{atm}}{\text{mol} \cdot \text{K}} \times 273.15 \text{ K}} = 1.428 \frac{\text{g}}{\text{L}}
\]

- Finding the molar mass of a volatile liquid
  - Weigh a flask with a known volume
  - Fill the flask with the vapors of the volatile liquid at a known temperature and pressure
  - Cool the flask and let the vapors condense
  - Reweigh the flask to get the mass of the vapors

**Example:** Calculate the molar mass of a liquid if 0.955 g of its vapors occupy 2.50 L at 200°C and 45.0 Torr.

\[
d = \frac{m}{V} = 0.955 \text{ g}/2.50 \text{ L} = 0.382 \text{ g/L}
\]

\[
T = 200^\circ \text{C} = 473 \text{ K}
\]

\[
P = 45.0 \text{ Torr} \times [1 \text{ atm}/760 \text{ Torr}] = 0.0592 \text{ atm}
\]

\[
M = \frac{dRT}{P} = \frac{0.382 \frac{\text{g}}{\text{L}} \times 0.08206 \frac{\text{L} \cdot \text{atm}}{\text{mol} \cdot \text{K}} \times 473 \text{ K}}{0.0592 \text{ atm}} = 250 \frac{\text{g}}{\text{mol}}
\]
Mixtures of Gasses
• Mixtures are treated just like pure gases – same gas laws apply
• Partial pressure of a gas in a mixture – the pressure the gas would exert if it occupied the container alone
• Dalton’s law of partial pressures – the total pressure ($P$) of a gaseous mixture is the sum of the partial pressures ($P_i$) of its components
  $$P = P_A + P_B + \ldots \text{ or } P = \sum P_i$$

• Mole fraction ($\chi_i$) of a gas in a mixture – a fraction of the total number of moles that belongs to that gas
  $$\chi_i = \frac{n_i}{\sum n_i} = \frac{n_i}{n} \quad \sum n_i = n \quad \sum \chi_i = 1$$
  The sum of all mol fractions is equal to one
  The ideal gas law can be written for each gas in a mixture in terms of partial pressures
  $$P_iV = n_iRT \quad PV = nRT$$

Example: Calculate the total pressure and the partial pressures of He and Ne in a 2.0 L mixture containing 1.0 g He and 2.0 g Ne at 20ºC.

grams of He and Ne $\rightarrow$ moles of He and Ne $\rightarrow$ mole fractions of He and Ne $\rightarrow$ partial pressures

$$1.0 \text{ g He} \times \left( \frac{1 \text{ mol He}}{4.00 \text{ g He}} \right) = 0.25 \text{ mol He}$$

$$2.0 \text{ g Ne} \times \left( \frac{1 \text{ mol Ne}}{20.18 \text{ g Ne}} \right) = 0.099 \text{ mol Ne}$$
\[ \chi_{He} = \frac{n_{He}}{n_{He} + n_{Ne}} = \frac{0.25}{0.25+0.099} = 0.72 \]
\[ \chi_{Ne} = \frac{n_{Ne}}{n_{He} + n_{Ne}} = \frac{0.099}{0.25+0.099} = 0.28 \]
\[ n = 0.25 + 0.099 = 0.35 \text{ mol} \]
\[ PV = nRT \quad \Rightarrow \quad P = \frac{nRT}{V} \]

- Collecting a gas over water
\[ P_{\text{total}} = P_{\text{gas}} + P_{\text{water}} \]
\[ P_{\text{water}} \text{ (vapor pressure of water)} \rightarrow \text{given in tables} \]

Example: A 2.5 L sample of \( \text{O}_2 \) gas was collected over water at 26°C and 745 torr atmospheric pressure. What is the mass of \( \text{O}_2 \) in the sample? (The vapor pressure of water at 26°C is 25 torr.)
\[ P_{\text{oxygen}} = P_{\text{total}} - P_{\text{water}} = 745 - 25 = 720 \text{ torr} \]
\[ T = 26 + 273.15 = 299 \text{ K} \]
\[ n_{\text{O}_2} = \frac{P_{\text{O}_2}V}{RT} = \frac{720 \text{ torr} \times \left( \frac{1 \text{ atm}}{760 \text{ torr}} \right) \times 2.5 \text{ L}}{0.08206 \text{ L} \cdot \text{atm} \cdot \text{mol}^{-1} \cdot \text{K}^{-1} \times 299 \text{ K}} = 0.097 \text{ mol} \]
\[ 0.097 \text{ mol O}_2 \times \left( \frac{32.0 \text{ g O}_2}{1 \text{ mol O}_2} \right) = 3.1 \text{ g O}_2 \]
Stoichiometry and the Ideal Gas Law

- The volume ratios of gases in reactions are the same as their mole ratios (follows from Avogadro’s principle)

\[ 3H_2(g) + N_2(g) \rightarrow 2NH_3(g) \]

\[ \Rightarrow 3 \text{ mol } H_2 \text{ react with } 1 \text{ mol } N_2 \]

\[ \Rightarrow 3 \text{ L } H_2 \text{ react with } 1 \text{ L } N_2 \]

**Example:** How many liters of N\(_2\) are needed to react completely with 5.0 L H\(_2\)?

\[ 5.0 \text{ L } H_2 \times \left[ \frac{1 \text{ L } N_2}{3 \text{ L } H_2} \right] = 1.7 \text{ L } N_2 \]

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**Example:** Calculate the volume of CO\(_2\) produced by the decomposition of 2.0 g CaCO\(_3\) at 25°C and 1.0 atm.

\[ \text{CaCO}_3(s) \rightarrow \text{CaO}(s) + \text{CO}_2(g) \]

\[ 2.0 \text{ g } \text{CaCO}_3 \times \left( \frac{1 \text{ mol } \text{CaCO}_3}{100.1 \text{ g } \text{CaCO}_3} \right) \times \left( \frac{1 \text{ mol } \text{CO}_2}{1 \text{ mol } \text{CaCO}_3} \right) = 0.020 \text{ mol } \text{CO}_2 \]

\[ V = \frac{nRT}{P} = \frac{0.020 \text{ mol} \times 0.08206 \text{ L} \cdot \text{atm}/\text{mol} \cdot \text{K} \times 298 \text{ K}}{1.0 \text{ atm}} = 0.49 \text{ L} \]

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**Example:** Calculate the mass of NaN\(_3\) needed to produce 10 L of N\(_2\) in an air bag at 25°C and 1.0 atm by the reaction:

\[ 6\text{NaN}_3(s) + 2\text{Fe}_2\text{O}_3(s) \rightarrow 3\text{Na}_2\text{O}_2(s) + 4\text{Fe}(s) + 9\text{N}_2(g) \]

\[ T = 298 \text{ K} \quad P = 1 \text{ atm} \quad V = 10 \text{ L} \quad n = ? \]

\[ n = \frac{PV}{RT} = \frac{1 \text{ atm} \times 10 \text{ L}}{0.08206 \frac{\text{L} \cdot \text{atm}}{	ext{mol} \cdot \text{K}} \times 298 \text{ K}} = 0.41 \text{ mol} \]

\[ 0.41 \text{ mol } \text{N}_2 \times \left( \frac{6 \text{ mol } \text{NaN}_3}{9 \text{ mol } \text{N}_2} \right) \left( \frac{65.02 \text{ g } \text{NaN}_3}{1 \text{ mol } \text{NaN}_3} \right) = 18 \text{ g } \text{NaN}_3 \]