### 5.5 The Kinetic Molecular Theory

- Postulates of the Kinetic Molecular Theory
- The gas particles are negligibly small (their volume can be neglected)
- The gas particles are in constant, random motion and move in straight lines until they collide
- The gas particles do not interact except during collisions. The collisions are elastic so there is no loss of energy due to friction
- The average kinetic energy of gas particles, $\overline{\boldsymbol{E}}_{\boldsymbol{k}}$, is proportional to the absolute temperature, $\boldsymbol{T}$
- A molecular view of the gas laws
- Charles's law ( $\boldsymbol{V} \propto \boldsymbol{T}$ )

- A molecular view of the gas laws
- Boyle's law $(V \propto 1 / P)$

- A molecular view of the gas laws
- Avogadro's law ( $\boldsymbol{V} \propto \boldsymbol{n}$ )

- Average kinetic energy of the gas particles

$$
\begin{aligned}
& \bar{E}_{k} \propto T \\
& \bar{E}_{k}=\frac{1}{2} m \overline{u^{2}} \\
\Rightarrow & \overline{u^{2}} \propto T
\end{aligned} \quad \sqrt{\overline{u^{2}}} \propto \sqrt{T}
$$

$\boldsymbol{m}$-mass of particles
$\overline{\boldsymbol{u}}^{2}$ - average square speed

- Root-mean-square speed of the gas particles

$$
\begin{gathered}
\boldsymbol{u}_{r m s}=\sqrt{\overline{\boldsymbol{u}^{2}}} \quad \boldsymbol{u}_{r m s} \propto \sqrt{\boldsymbol{T}} \\
\boldsymbol{u}_{r m s}=\sqrt{\frac{\mathbf{3 R T}}{M}}
\end{gathered}
$$

## The Maxwell Distribution of Speeds

- Gives the fraction of particles moving at each particle speed
- Gas molecules travel with a wide range of speeds with a bell-shaped distribution
- The most probable speed, the average speed and the root-mean-square speed are very close in magnitude

Example: Calculate the root-mean-square speed of $\mathrm{N}_{2}$ at $25^{\circ} \mathrm{C}$.

$$
\begin{aligned}
& T=25^{\circ} \mathrm{C}=298 \mathrm{~K} \\
& M=28.02 \mathrm{~g} / \mathrm{mol}=0.02802 \mathrm{~kg} / \mathrm{mol} \\
& R=8.314 \mathrm{~J} / \mathrm{mol} \cdot \mathrm{~K}
\end{aligned}
$$

$$
u_{r m s}=\sqrt{\frac{3 R T}{M}}=\sqrt{\frac{3 \times 8.314 \frac{\mathrm{~J}}{\mathrm{~mol} \cdot \mathrm{~K}} \times 298 \mathrm{~K}}{0.02802 \frac{\mathrm{~kg}}{\mathrm{~mol}}}}=515 \sqrt{\frac{\mathrm{~J}}{\mathrm{~kg}}}
$$

$$
515 \sqrt{\frac{\mathrm{~kg} \cdot \mathrm{~m}^{2} / \mathrm{s}^{2}}{\mathrm{~kg}}}=515 \sqrt{\frac{\mathrm{~m}^{2}}{\mathrm{~s}^{2}}}=515 \frac{\mathrm{~m}}{\mathrm{~s}}
$$



- The range of speeds widens and $\boldsymbol{u}_{\boldsymbol{r m s}}$ increases with increasing the temperature


Molecular speed at a given $T$

- The range of speeds widens and $\boldsymbol{u}_{r m s}$ increases with decreasing the molar mass of the gas
- Lighter gases have higher molecular speeds


## Diffusion and Effusion

- Diffusion - gradual dispersal of one substance through another
- gases diffuse from places with high to places with low concentration
- Effusion - escape of a substance through a small hole into vacuum
- effusion through porous materials, pin holes, cracks, etc.
- The meaning of temperature

$$
\begin{gathered}
\bar{E}_{k}=\frac{1}{2} m \bar{u}^{2}=\frac{1}{2} m u_{r m s}^{2}=\frac{1}{2} m\left(\frac{3 R T}{M}\right)=\frac{m 3 R T}{2 m N_{A}} \\
\bar{E}_{k}=\frac{3 R T}{2 N_{A}} \quad \text { and } \quad u_{r m s}=\sqrt{\frac{3 R T}{M}}
\end{gathered}
$$

$\Rightarrow u_{r m s}$ depends on $T$ and $M$
$\Rightarrow \bar{E}_{k}$ depends only on $T$
$\Rightarrow T$ is a measure of the average kinetic energy of the molecular motion

- Graham's Law - the effusion rate $(\boldsymbol{E R})$ of a gas is inversely proportional to the square root of its molar mass (same relation is valid in general for the diffusion rate)

$$
E R \propto \sqrt{\frac{1}{M}}
$$

- Can be explained with $\boldsymbol{u}_{\boldsymbol{r m s}}=\sqrt{ } \mathbf{3} \boldsymbol{R} \boldsymbol{T} / \boldsymbol{M}$
- The time of effusion $\left(\boldsymbol{t}_{\text {eff }}\right)$ is inversely proportional to $\boldsymbol{E R}$

$$
t_{e f f} \propto \frac{1}{E R} \Rightarrow t_{e f f} \propto \sqrt{M}
$$

For two gases, A and B:
$\frac{\boldsymbol{E R}(A)}{\boldsymbol{E R}(B)}=\sqrt{\frac{M_{B}}{M_{A}}} \quad \frac{\boldsymbol{t}_{\text {eff }}(A)}{\boldsymbol{t}_{\text {eff }}(B)}=\sqrt{\frac{M_{A}}{M_{B}}}$
Example: If it takes a certain amount of $\mathrm{H}_{2}$ 15 s to effuse through a small hole, how long does it take for the same amount of $\mathrm{O}_{2}$ ?
$\frac{t_{e f f}\left(O_{2}\right)}{t_{e f f}\left(H_{2}\right)}=\sqrt{\frac{M_{O_{2}}}{M_{H_{2}}}} \quad t_{e f f}\left(O_{2}\right)=\sqrt{\frac{M_{O_{2}}}{M_{H_{2}}}} \times t_{e f f}\left(H_{2}\right)$
$t_{e f f}\left(O_{2}\right)=\sqrt{\frac{32.00 \mathrm{~g} / \mathrm{mol}}{2.02 \mathrm{~g} / \mathrm{mol}}} \times 15 \mathrm{~s}=60 \mathrm{~s}$


### 5.6 Real Gases

- Real gases deviate from ideal behavior
- Compression factor ( $\boldsymbol{Z}$ )

$$
Z=P V / n R T
$$

- For ideal gases:

$$
P V=n R T \quad \Rightarrow \quad Z=P V / n R T=1
$$

- A plot of $\boldsymbol{Z}$ versus $\boldsymbol{P}$ gives a straight line for ideal gases, but not for real gases
- Negative deviations - important at moderately high pressures $(\boldsymbol{P V} / \boldsymbol{n} \boldsymbol{R} \boldsymbol{T}<\mathbf{1})$
- Due to attractive forces between the molecules
- The molecules attract each other and impact the walls with a weaker force ( $\boldsymbol{P}$ and $\boldsymbol{Z}$ decrease)
- Positive deviations - important at very high pressures $(\boldsymbol{P V} / \boldsymbol{n R T} \boldsymbol{P} \mathbf{1})$
- Due to the actual volume of the molecules
- The physical volume of the molecules reduces the free volume in the container, but we still use the volume of the entire container, $\boldsymbol{V}$, which is larger than the free volume ( $\boldsymbol{Z}$ increases)
- Van der Waals equation:

$$
\left(P+\frac{a n^{2}}{V^{2}}\right)(V-n b)=n R T
$$

$\boldsymbol{a}, \boldsymbol{b}$ - van der Waals constants (zero for ideal gases) $\boldsymbol{a} \boldsymbol{n}^{2} / \boldsymbol{V}^{2}$ - pressure correction ( $\boldsymbol{a}$ depends on the attractive forces between molecules)
$\boldsymbol{n} \boldsymbol{b}$ - volume correction ( $\boldsymbol{b}$ is a measure for the actual volume of the gas molecules)

- Real gases approach ideal behavior at low pressures and high temperatures (away from conditions of condensation)

