The Bohr model of the H atom (1913)

- Explains the hydrogen atomic emission spectrum by using the idea of quantization
- **Postulates:**
  - The electron travels around the nucleus in **circular orbits** without loss of energy
  - The angular momentum of the electron is quantized → **only certain orbits are allowed**
- **Consequences:**
  - The energy of the H atom is quantized → only certain **discrete energy levels** (stationary states) are allowed
  - Each circular orbit corresponds to one ***E***-level

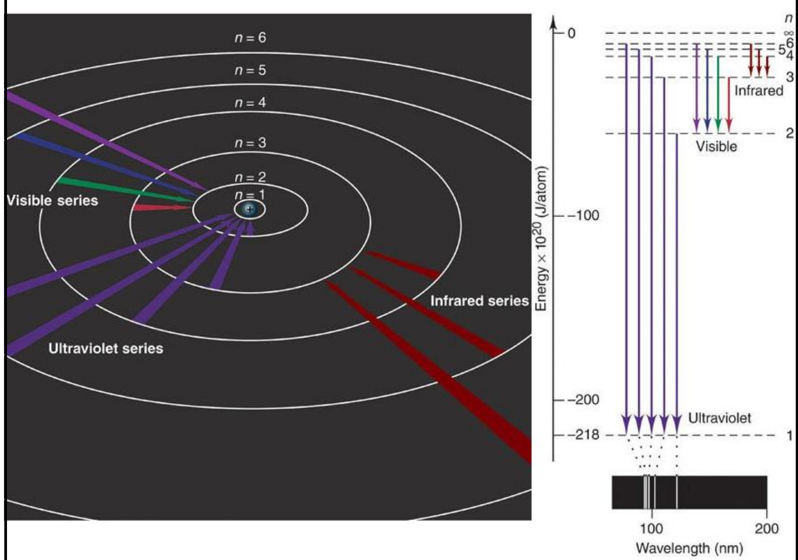
- **Consequences** (cont.):
  - A transition between two energy states generates a **photon** with energy equal to the difference between the two levels (ΔE)

$$E_{ph} = E_{state\ 2} - E_{state\ 1} = h\nu \quad \Rightarrow \quad \Delta E = h\nu$$

  - A photon with a specific (discrete) frequency is emitted for each transition from a higher to a lower ***E***-level

⇒**Atomic emission spectra consist of discrete lines**

  - Each orbit is labeled with a number, ***n***, starting from the orbit closest to the nucleus (***n*** = 1, 2, …)
  - The same number is used to label the energy levels → ***n*** is the **quantum number**



- **Energy states of the H atom**

$$E_n = -B\left(\frac{Z^2}{n^2}\right) \qquad n = 1, 2, 3, \ldots, \infty$$

B is a constant ($B = 2.18\times10^{-18}$ **J**)

Z is the nuclear charge (**For H:** $Z = 1 \rightarrow E_n = -B/n^2$)

- **Ground state** – the lowest energy state (***n*** = 1)

$$E_1 = -B/1^2 = -B = -2.18\times10^{-18}\ \text{J}$$

- **Excited states** – higher energy levels (***n*** > 1)
  - The energy increases with increasing ***n***
  - The highest possible energy is for ***n*** = ∞ (the electron is completely separated from the nucleus)

$$E_\infty = -B/\infty^2 = 0$$

- A transition between two E-levels with quantum numbers n_1 and n_2 will produce a photon with energy equal to the E-difference between the levels, ΔE:

$$\Delta E = E_{n_2} - E_{n_1} = \left(-\frac{B}{n_2^2}\right) - \left(-\frac{B}{n_1^2}\right) = B\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$$\Delta E = E_{ph} = h\nu = \frac{hc}{\lambda} = B\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$$\Rightarrow \frac{1}{\lambda} = \frac{B}{hc}\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) \leftarrow \textbf{Rydberg eq.} \left(\frac{B}{hc} = R\right)$$

- **Ionization energy (I)** of the H atom – the energy needed to completely remove the electron from a H atom in its ground state (can be viewed as the energy change from E_1 to E_∞)

$$\Delta E = E_\infty - E_1 = \left(-\frac{B}{\infty^2}\right) - \left(-\frac{B}{1^2}\right) = 0 - (-B) = B$$

$$\Rightarrow I = B = 2.18 \times 10^{-18}\ \text{J/atom}$$

- **Limitations of the Bohr Model**
  - Applicable only to **H-like** atoms and ions (having a single electron) in the absence of strong electric or magnetic fields (H, He^+, Li^{2+}, …)

7.3 The Wave-Particle Duality of Matter and Energy

- **Mass-energy equivalency** (Einstein)

$$E = mc^2$$

- For a photon with energy $E = h\nu = hc/\lambda$:

$$E = mc^2 = hc/\lambda \;\Rightarrow\; mc = h/\lambda \;\Rightarrow\; \lambda = h/mc$$

$$\Rightarrow \lambda = h/p \qquad p \text{ – photon momentum}$$

  - The equation shows that the wave-like photons have particle-like mass and momentum
- Experimental evidence (Compton, 1923)

- **De Broglie's hypothesis** (1924) – all matter has wave-like properties (just as waves have particle-like properties)
  - For a particle with mass, m, and velocity, u, the wavelength is:

$$\lambda = h/mu$$

  - De Broglie's equation is equivalent to that for a photon ($\lambda = h/mc$)
  - De Broglie's equation combines particle properties (m, u) with wave properties (λ)

$\Rightarrow$ Matter and energy exhibit wave-particle duality

- **Example:** Calculate the wavelengths of an electron (***m*** **= 9.109×10^{-31} kg**) with velocity **2.2×10^{6} m/s** and a bullet (***m*** **= 5.0 g**) traveling at **700. m/s**.

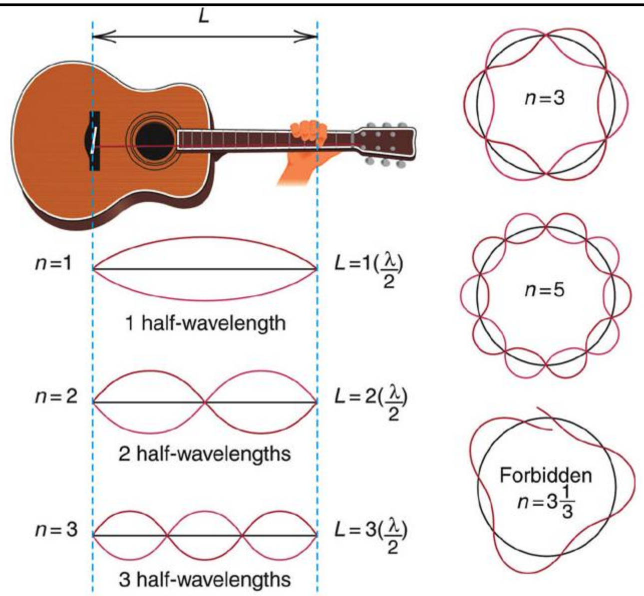
$$\lambda(e^-) = \frac{h}{mu} = \frac{6.626\times10^{-34}\,\mathrm{J\cdot s}}{9.109\times10^{-31}\mathrm{kg}\times2.2\times10^{6}\,\mathrm{m/s}}$$

$= 3.3\times10^{-10}\,\mathrm{m} = 0.33\ \mathrm{nm}$ → **comparable to atomic sizes**

$$\lambda(bul) = \frac{h}{mu} = \frac{6.626\times10^{-34}\,\mathrm{J\cdot s}}{5.0\times10^{-3}\mathrm{kg}\times700.\ \mathrm{m/s}}$$

$= 1.9\times10^{-34}\,\mathrm{m}$ → **very short, undetectable**

- **Experimental evidence** (Davisson and Germer, 1927)
  - **Diffraction of electrons** by crystal surfaces
  - Diffraction patterns are consistent with the wavelength predicted by de Broglie's relation
- The electron can be treated as a wave with a very short wavelength (similar to the wavelength of x-rays)
- The electron confined in the **H** atom can be treated as a **standing wave** having discrete frequencies (energies) like a guitar string



- **Heisenberg's uncertainty principle** (1927) – the exact position and momentum (velocity) of a particle can not be known simultaneously

$$\Delta x\cdot\Delta p \geq h/4\pi$$

Δx *and* ***Δp*** **=** ***mΔu*** – uncertainty in position and momentum, respectively
  - A consequence of the wave-particle duality of matter
  - The exact location of very small particles is not well known due to their wave-like properties
  - The probability to find a particle at a particular location depends on the amplitude (intensity) of the wave at this location