1.5 Uncertainty of Measurements

- Represents the reliability of measurements
- Reported as: **number ± uncertainty** (4.88 ± 0.05 kg)
- If not reported: assume **±1 in the last reported digit** (3.7 cm → 3.7 ± 0.1 cm)
- Exact numbers – no uncertainty (5 tables, 10 apples, 1 min = 60 s, 1 in = 2.54 cm)

- **Significant figures** – digits of a number known with some degree of certainty
  - All non-zero digits
  - All zeros after the first non-zero digit
  - Exception – trailing zeros in numbers without decimal point are not significant
- More significant figures ↔ less uncertainty

Examples:

1.32 → 3 sf

0.005030 → 4 sf

4500 → 2 sf

4500. → 4 sf

- Scientific notation – representation in the form → $A \times 10^a$
  - A → a decimal number between 1 and 10
  - a → a positive or negative integer
- **Examples:**

  $0.00134 = 1.34 \times 10^{-3}$

  $134 = 1.34 \times 10^{2}$

  - all digits in A are significant

- Examples of significant figures

Decimal notation	Scientific notation	Number of sf
0.751	7.51×10^{-1}	3
0.007 51	7.51×10^{-3}	3
0.070 51	7.051×10^{-2}	4
0.750 100	$7.501\,00 \times 10^{-1}$	6
7.5010	7.5010	5
7501	7.501×10^{2}	4
7500	7.5×10^{3}	2*
7500.	7.500×10^{3}	4

- **Significant figures in calculations**
  - Rounding off (only at the end of a calculation)
    - round up, if next digit is **above 5**
    - round down, if next digit is **below 5**
    - round to the nearest even number, if next digit is **equal to 5** and it is the last nonzero digit of the number (if 5 is not the last nonzero digit, round up)

Examples: Round to 3 sf.

3.7643 → 3.76

3.7683 → 3.77

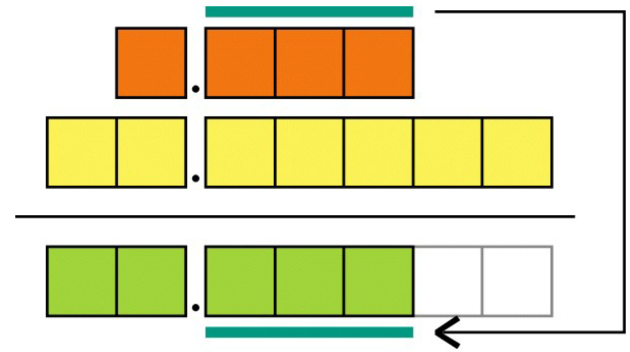
3.7653 → 3.77

3.765 → 3.76

3.765 → 3.76

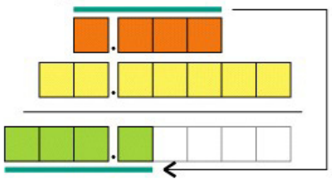
3.755 → 3.76

- Addition and subtraction
  - the number of decimal places in the result is the same as the smallest number of decimal places in the data

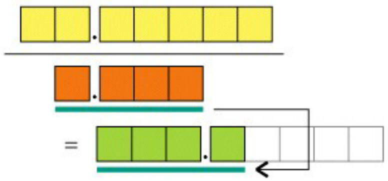


- Multiplication and division
  - the number of significant figures in the result is the same as the smallest number of significant figures in the data

Multiplication



Division



Examples:

0.0354 + 12.1 = 12.1 ← (12.1354)

5.7×0.0651 = 0.37 ← (0.37107)

5.7/0.0651 = 88 ← (87.55760369)

3.568 in × (2.54 cm/1 in) = 9.063 cm

- **Precision and accuracy**
  - Two aspects of uncertainty
- **Precision** – agreement among repeated measurements
  - **Random error** – deviation from the average in a series of repeated measurements (some values higher, some values lower than the average)

small random error ↔ high precision

high precision ↔ more sf in the result

- **Accuracy** – agreement of a measurement with the true or accepted value
  - **Systematic error** – deviation of the average from the true value (present in the whole set of measurements – either all high or all low)

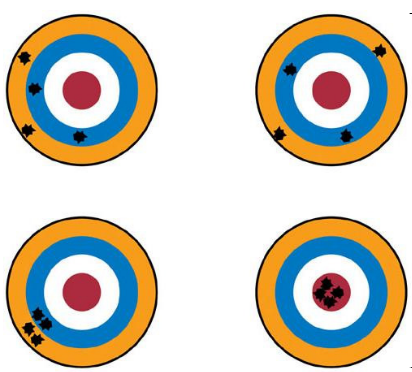
small systematic error ↔ high accuracy

- **Instrument calibration** – comparison with a known standard
  - Essential for avoiding systematic error

- Examples of precision and accuracy

Low precision
Low accuracy

Low precision
High accuracy



High precision
Low accuracy

High precision
High accuracy

Example:

- A car is moving at exactly **60 mi/hr**. Compare the precision and accuracy of the following two series of speed measurements using two different radars.

A → 61.5, 58.3, 62.7, 63.5, 57.1 (average 60.6)

B → 62.0, 62.5, 61.8, 62.2, 62.1 (average 62.1)

A → more accurate, less precise

B → less accurate, more precise