### 18.2 Autoionization of Water and $\boldsymbol{p H}$

- Water can act as both proton donor (acid) and proton acceptor (base) $\rightarrow$ amphoteric

- Ion-product constant of water $\left(\boldsymbol{K}_{w}\right)$
$K_{c}=\frac{\left[\mathrm{H}_{3} \mathrm{O}^{+}\right]\left[\mathrm{OH}^{-}\right]}{\left[\mathrm{H}_{2} \mathrm{O}\right]^{2}}$ $\rightarrow\left[\mathbf{H}_{2} \mathbf{O}\right]$ can be assumed constant and combined with $\boldsymbol{K}_{c}$
$\Rightarrow K_{w}=\left[\mathrm{H}_{3} \mathrm{O}^{+}\right]\left[\mathrm{OH}^{-}\right] \rightarrow K_{w}=K_{c}\left[\mathrm{H}_{2} \mathbf{O}\right]^{2}$
$\boldsymbol{K}_{w} \rightarrow$ ion-product (autoprotolysis) constant of water
$>$ At $25^{\circ} \mathrm{C}, \boldsymbol{K}_{w}=\mathbf{1 . 0 \times 1 0 ^ { - 1 4 }}$
$>$ In pure water $\rightarrow\left[\mathrm{H}_{3} \mathrm{O}^{+}\right]=\left[\mathrm{OH}^{-}\right]=\boldsymbol{x}$

$$
\begin{aligned}
& \rightarrow \boldsymbol{K}_{w}=\boldsymbol{x}^{2} \rightarrow \boldsymbol{x}=\sqrt{ } \boldsymbol{K}_{w}=\sqrt{ } 1.0 \times 10^{-14}=1.0 \times 10^{-7} \\
& \quad \Rightarrow\left[\mathbf{H}_{3} \mathbf{O}^{+}\right]=\left[\mathbf{O H}^{-}\right]=\mathbf{1 . 0} \mathbf{0} \mathbf{1 0 ^ { - 7 }} \mathbf{M}
\end{aligned}
$$

$>$ In water solutions $\rightarrow\left[\mathrm{H}_{3} \mathrm{O}^{+}\right]$and $\left[\mathrm{OH}^{-}\right]$are inversely proportional to each other
$\rightarrow K_{w}=\left[\mathrm{H}_{3} \mathrm{O}^{+}\right]\left[\mathrm{OH}^{-}\right] \Rightarrow \uparrow\left[\mathrm{H}_{3} \mathrm{O}^{+}\right] \Leftrightarrow \downarrow\left[\mathrm{OH}^{-}\right]$
$\Rightarrow \mathrm{H}_{3} \mathrm{O}^{+}$and $\mathrm{OH}^{-}$are present in different
proportions in both acid and base solutions
Neutral solutions $\rightarrow \quad\left[\mathbf{H}_{3} \mathbf{O}^{+}\right]=\left[\mathrm{OH}^{-}\right]$
Acidic solution $\rightarrow \quad\left[\mathrm{H}_{3} \mathrm{O}^{+}\right]>\left[\mathrm{OH}^{-}\right]$
Basic solutions $\rightarrow\left[\mathrm{H}_{3} \mathrm{O}^{+}\right]<\left[\mathrm{OH}^{-}\right]$

## The pH Scale

- The pH scale is a logarithmic scale

$$
\begin{gathered}
\boldsymbol{p H}=-\log \left[\mathbf{H}_{\mathbf{3}} \mathbf{O}^{+}\right] \\
\rightarrow \log \left[\mathrm{H}_{3} \mathrm{O}^{+}\right]=-\boldsymbol{p H} \quad \Rightarrow\left[\mathbf{H}_{3} \mathbf{O}^{+}\right]=\mathbf{1 0}^{-\boldsymbol{p H}}
\end{gathered}
$$

$$
\uparrow\left[\mathrm{H}_{3} \mathrm{O}^{+}\right] \Leftrightarrow \downarrow p H
$$

$\rightarrow$ If $\left[\mathrm{H}_{3} \mathrm{O}^{+}\right]=\left[\mathrm{OH}^{-}\right]=1 \times 10^{-7} \mathrm{M}$ (neutral solution)
$\rightarrow \boldsymbol{p} \boldsymbol{H}=-\log \left(1 \times 10^{-7}\right) \rightarrow$ neutral solution $\rightarrow \boldsymbol{p H}=\mathbf{7}$
$\rightarrow$ If $\left[\mathrm{H}_{3} \mathrm{O}^{+}\right]>1 \times 10^{-7} \mathrm{M} \rightarrow$ acidic solution $\rightarrow \boldsymbol{p H}<7$
$\rightarrow$ If $\left[\mathrm{H}_{3} \mathrm{O}^{+}\right]<1 \times 10^{-7} \mathrm{M} \rightarrow$ basic solution $\rightarrow \mathrm{pH}>7$
Examples: $\left[\mathrm{H}_{3} \mathrm{O}^{+}\right]=1 \rightarrow \boldsymbol{p H}=-\log 1=\mathbf{0}$

$$
\left[\mathrm{H}_{3} \mathrm{O}^{+}\right]=10^{-14} \rightarrow \boldsymbol{p} \boldsymbol{H}=-\log 10^{-14}=\mathbf{1 4}
$$

- The $p$-notation $\rightarrow p X=-\log X$

$$
\mathrm{pOH}=-\log \left[\mathrm{OH}^{-}\right]
$$

$\rightarrow \log \left[\mathrm{OH}^{-}\right]=-\mathbf{p O H} \Rightarrow\left[\mathrm{OH}^{-}\right]=\mathbf{1 0}^{-\boldsymbol{p o H}}$

$$
p K_{w}=-\log K_{w}
$$

$\rightarrow\left[\mathrm{H}_{3} \mathrm{O}^{+}\right]\left[\mathrm{OH}^{-}\right]=\boldsymbol{K}_{w} \leftarrow$ (take a -log)
$\rightarrow-\log \left[\mathrm{H}_{3} \mathrm{O}^{+}\right]+\left(-\log \left[\mathrm{OH}^{-}\right]\right)=-\log \boldsymbol{K}_{w}$
$\Rightarrow \mathrm{pH}+\mathrm{pOH}=\mathrm{p} \mathrm{K}_{w}$
$\rightarrow$ At $25^{\circ} \mathrm{C}, \boldsymbol{K}_{w}=\mathbf{1 . 0 \times 1 0 ^ { - 1 4 }} \rightarrow \boldsymbol{p} K_{w}=14.00$
$\Rightarrow \mathrm{pH}+\mathrm{pOH}=14.00$
$\rightarrow$ If $\boldsymbol{p H}=7 \rightarrow \boldsymbol{p O H}=7 \rightarrow$ neutral solution
$\rightarrow$ Acidic ( $\left.\downarrow_{\mathrm{pH}}, \uparrow \mathrm{pOH}\right) ; \quad$ Basic $\left(\uparrow \mathrm{pH}, \downarrow_{\mathrm{pOH}}\right)$

| $\frac{0}{6}$ |  | $\left[\mathrm{H}_{3} \mathrm{O}^{+}\right]$ | pH | [ $\mathrm{OH}^{-}$] | pOH |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | BASIC | $1.0 \times 10^{-15}$ | 15.00 | $1.0 \times 10^{1}$ | -1.00 |
|  |  | $1.0 \times 10^{-14}$ | 14.00 | $1.0 \times 10^{0}$ | 0.00 |
|  |  | $1.0 \times 10^{-13}$ | 13.00 | $1.0 \times 10^{-1}$ | 1.00 |
|  |  | $1.0 \times 10^{-12}$ | 12.00 | $1.0 \times 10^{-2}$ | 2.00 |
|  |  | $1.0 \times 10^{-11}$ | 11.00 | $1.0 \times 10^{-3}$ | 3.00 |
|  |  | $1.0 \times 10^{-10}$ | 10.00 | $1.0 \times 10^{-4}$ | 4.00 |
|  |  | $1.0 \times 10^{-9}$ | 9.00 | $1.0 \times 10^{-5}$ | 5.00 |
|  |  | $1.0 \times 10^{-8}$ | 8.00 | $1.0 \times 10^{-6}$ | 6.00 |
|  | NEUTR | $1.0 \times 10^{-7}$ | 7.00 | $1.0 \times 10^{-7}$ | 7.00 |
|  | ACIDIC | $1.0 \times 10^{-6}$ | 6.00 | $1.0 \times 10^{-8}$ | 8.00 |
|  |  | $1.0 \times 10^{-5}$ | 5.00 | $1.0 \times 10^{-9}$ | 9.00 |
|  |  | $1.0 \times 10^{-4}$ | 4.00 | $1.0 \times 10^{-10}$ | 10.00 |
|  |  | $1.0 \times 10^{-3}$ | 3.00 | $1.0 \times 10^{-11}$ | 11.00 |
|  |  | $1.0 \times 10^{-2}$ | 2.00 | $1.0 \times 10^{-12}$ | 12.00 |
|  |  | $1.0 \times 10^{-1}$ | 1.00 | $1.0 \times 10^{-13}$ | 13.00 |
|  |  | $1.0 \times 10^{0}$ | 0.00 | $1.0 \times 10^{-14}$ | 14.00 |
|  |  | $1.0 \times 10^{1}$ | -1.00 | $1.0 \times 10^{-15}$ | 15.00 |

Example: A household ammonia solution has a $\boldsymbol{p H}$ of 11.92. What is the $\boldsymbol{p O H}$ and the concentration of $\mathrm{OH}^{-}$in it?
$\rightarrow p O H=14.00-p H=14.00-11.92=2.08$
$\rightarrow\left[\mathrm{OH}^{-}\right]=10^{-\mathrm{pOH}}=10^{-2.08}=8.3 \times 10^{-3}$
Note: The \# of sig. figs in $\left[\mathrm{OH}^{-}\right]$(or $\left[\mathrm{H}_{3} \mathrm{O}^{+}\right]$) equals the number of decimal places in $\mathbf{p O H}$ ( or $\boldsymbol{p H}$ )

- Measuring $\boldsymbol{p H}$ in the laboratory
- Indicators - have different colors at different $\boldsymbol{p H}$ ( $\boldsymbol{p} \boldsymbol{H}$ paper, indicator solutions)
$-\boldsymbol{p H}$ meters - measure the $\boldsymbol{p H}$ by measuring the potential of a $\boldsymbol{p H}$ sensitive electrode immersed in the solution (more accurate)
$K_{a}$ and $K_{b}$ of a Conjugate Acid-Base Pair
- In a solution of the weak acid HA, both HA and its conjugate base, $\mathbf{A}^{-}$, react with water

$$
\begin{aligned}
& \mathrm{HA}+\mathrm{H}_{2} \mathrm{O} \leftrightarrow \mathrm{~A}^{-}+\mathrm{H}_{3} \mathrm{O}^{+} \quad \boldsymbol{K}_{\boldsymbol{a}} \text { (of HA) } \\
& \mathrm{A}^{-}+\mathrm{H}_{2} \mathrm{O} \leftrightarrow \mathrm{HA}+\mathrm{OH}^{-} \quad \boldsymbol{K}_{\boldsymbol{b}} \text { (of A }{ }^{-} \text {) } \\
& \oplus \rightarrow \mathrm{H}_{2} \mathrm{O}+\mathrm{H}_{2} \mathrm{O} \leftrightarrow \mathrm{H}_{3} \mathrm{O}^{+}+\mathrm{OH}^{-} \quad \boldsymbol{K}_{\boldsymbol{w}}=\boldsymbol{K}_{\boldsymbol{a}} \times \boldsymbol{K}_{\boldsymbol{b}} \\
& \boldsymbol{K}_{a} \times \boldsymbol{K}_{b}=\frac{\left[\mathrm{H}_{3} \mathbf{O}^{+}\right]\left[\mathrm{A}^{-}\right]}{[\mathbf{H A}]} \times \frac{[\mathrm{HA}]\left[\mathrm{OH}^{-}\right]}{\left[\mathrm{A}^{-}\right]}=\left[\mathbf{H}_{3} \mathbf{O}^{+}\right]\left[\mathrm{OH}^{-}\right]=\boldsymbol{K}_{w} \\
& \Rightarrow \text { For a conjugate acid-base pair } \rightarrow \boldsymbol{K}_{a} \times \boldsymbol{K}_{\boldsymbol{b}}=\boldsymbol{K}_{\boldsymbol{w}} \\
& -\log \boldsymbol{K}_{\boldsymbol{a}}+\left(-\log \boldsymbol{K}_{\boldsymbol{b}}\right)=-\log \boldsymbol{K}_{\boldsymbol{w}} \\
& \Rightarrow p \boldsymbol{K}_{a}+\boldsymbol{p} \boldsymbol{K}_{b}=\boldsymbol{p} \boldsymbol{K}_{w} \rightarrow \mathrm{At} 25^{\circ} \mathrm{C}, p \boldsymbol{K}_{a}+\boldsymbol{p} \boldsymbol{K}_{b}=\mathbf{1 4 . 0 0}
\end{aligned}
$$



## Example:

Given $\boldsymbol{K}_{\boldsymbol{a}}(\mathrm{HF})=6.8 \times 10^{-4}$ and $\boldsymbol{K}_{\boldsymbol{a}}(\mathrm{HCN})=6.2 \times 10^{-10}$, determine the preferred direction of the reaction

$$
\mathrm{F}^{-}+\mathrm{HCN} \leftrightarrow \mathrm{HF}+\mathrm{CN}^{-}
$$

$\rightarrow$ Compare the strengths of the acids on both sides $\rightarrow$ HF is stronger than HCN (HF has higher $\boldsymbol{K}_{a}$ value)
$\rightarrow$ Compare the strengths of the bases on both sides
$\rightarrow$ Since HF is a stronger acid than HCN , the conjugate base of $\mathrm{HF}, \mathrm{F}^{-}$, is a weaker base than the conjugate base of $\mathrm{HCN}, \mathrm{CN}^{-}$
$\rightarrow \mathrm{CN}^{-}$is stronger than $\mathrm{F}^{-}$( $\mathrm{CN}^{-}$has higher $\boldsymbol{K}_{\boldsymbol{b}}$ value)
$\Rightarrow \mathrm{F}^{-}+\mathrm{HCN} \leftrightarrow \mathrm{HF}+\mathrm{CN}^{-}$is shifted to the left $\left(\boldsymbol{K}_{\boldsymbol{c}}<1\right)$

### 18.3 Relative Acid-Base Strength and the Net Direction of Reaction

- An acid-base reaction proceeds to a greater extent from the stronger acid and stronger base towards the weaker acid and weaker base
$\mathrm{HA}+\mathrm{B} \leftrightarrow \mathrm{A}^{-}+\mathrm{HB}^{+}$
stronger acid + stronger base $\rightarrow$ weaker base + weaker acid $\rightarrow$ The reaction is shifted towards A- and $\mathrm{HB}^{+}\left(\boldsymbol{K}_{\boldsymbol{c}}>1\right)$


## Examples:



