## Ionic Equilibria in Aqueous Systems

### 19.1 Acid-Base Buffers

- Resist changes in $\boldsymbol{p} \boldsymbol{H}$ upon addition of acids $\left(\mathrm{H}^{+}\right)$ or bases $\left(\mathrm{OH}^{-}\right)$


## The Buffer Action

- Buffer solutions consist of to components
- Acid component - reacts with added bases $\left(\mathrm{OH}^{-}\right)$
- Base component - reacts with added acids $\left(\mathrm{H}^{+}\right)$
- The two components must be a conjugate acidbase pair
- The two components must be present in high concentrations


## The $\boldsymbol{p H}$ of a Buffer

- For a weak acid buffer (HA/A-), A supplied in solution by means of the salt, MA

$$
\mathrm{MA} \rightarrow \mathrm{M}^{+}+\mathrm{A}^{-}
$$

- HA and $\mathrm{A}^{-}$reach equilibrium

$$
\mathrm{HA}+\mathrm{H}_{2} \mathrm{O} \leftrightarrow \mathrm{H}_{3} \mathrm{O}^{+}+\mathrm{A}^{-}
$$

$$
K_{a}=\frac{\left[\mathbf{H}_{3} \mathbf{O}^{+}\right]\left[\mathrm{A}^{-}\right]}{[\mathrm{HA}]} \Rightarrow\left[\mathrm{H}_{3} \mathbf{O}^{+}\right]=K_{a} \frac{[\mathbf{H A}]}{\left[\mathrm{A}^{-}\right]}
$$

$\Rightarrow$ The $\left[\mathrm{H}_{3} \mathrm{O}^{+}\right]$and $\boldsymbol{p} \boldsymbol{H}$ depend on the ratio $[\mathbf{H A}] /\left[\mathbf{A}^{-}\right]$ $\rightarrow$ If [ HA ] and $\left[\mathrm{A}^{-}\right]$are relatively large, adding small amounts of $\mathrm{H}_{3} \mathrm{O}^{+}$or $\mathrm{OH}^{-}$changes the ratio only slightly, so $\left[\mathrm{H}_{3} \mathrm{O}^{+}\right] \approx$ constant
$>$ Weak acid buffer - a solution of the weak acid,
HA , and its conjugate base, $\mathrm{A}^{-}\left(\mathrm{HF} / \mathrm{F}^{-}, \ldots\right)$
$>$ Add $\mathrm{H}_{3} \mathrm{O}^{+}($acid $) \rightarrow\left(\mathrm{A}^{-}\right.$consumes the added $\left.\mathrm{H}_{3} \mathrm{O}^{+}\right)$

$$
\mathrm{H}_{3} \mathrm{O}^{+}+\mathrm{A}^{-} \rightarrow \mathrm{H}_{2} \mathrm{O}+\mathrm{HA}
$$

$>$ Add $\mathrm{OH}^{-}($base $) \rightarrow\left(\mathrm{HA}^{-}\right.$consumes the added $\left.\mathrm{OH}^{-}\right)$

$$
\mathrm{OH}^{-}+\mathrm{HA} \rightarrow \mathrm{H}_{2} \mathrm{O}+\mathrm{A}^{-}
$$

$>$ Weak base buffer - a solution of the weak base, B, and its conjugate acid, $\mathrm{HB}^{+}\left(\mathrm{NH}_{4}^{+} / \mathrm{NH}_{3}, \ldots\right)$
$>$ Add $\mathrm{H}_{3} \mathrm{O}^{+}($acid $) \rightarrow\left(\mathrm{B}\right.$ consumes the added $\left.\mathrm{H}_{3} \mathrm{O}^{+}\right)$

$$
\mathrm{H}_{3} \mathrm{O}^{+}+\mathrm{B} \rightarrow \mathrm{H}_{2} \mathrm{O}+\mathrm{HB}^{+}
$$

$>$ Add $\mathrm{OH}^{-}($base $) \rightarrow\left(\mathrm{HB}^{+}\right.$consumes the added $\left.\mathrm{OH}^{-}\right)$ $\mathrm{OH}^{-}+\mathrm{HB}^{+} \rightarrow \mathrm{H}_{2} \mathrm{O}+\mathrm{B}$
$\Rightarrow$ The addition of $\mathrm{H}_{3} \mathrm{O}^{+}$and $\mathrm{OH}^{-}$changes the relative amounts of the buffer components (HA/A- or $\mathrm{HB}^{+} / \mathrm{B}$ )

- Both HA and $\mathrm{A}^{-}$react with water

1. $\mathrm{HA}+\mathrm{H}_{2} \mathrm{O} \leftrightarrow \mathrm{H}_{3} \mathrm{O}^{+}+\mathrm{A}^{-}$
2. $\mathrm{A}^{-}+\mathrm{H}_{2} \mathrm{O} \leftrightarrow \mathrm{OH}^{-}+\mathrm{HA}$
$\rightarrow$ Both equilibria are shifted to the left, so the amounts of HA and A- produced are often very small compared to the total concentrations, $C_{H A}$ and $C_{A^{-}}$ $\Rightarrow$ Assume that $[\mathrm{HA}]_{2}$ and $\left[\mathrm{A}^{-}\right]_{1} \ll C_{H A}$ and $C_{A^{-}}$ $[\mathrm{HA}]=C_{H A}-\left[\mathrm{A}^{-}\right]_{1}+\left[\mathrm{HA}_{2} \approx C_{H A}=\boldsymbol{C}_{\boldsymbol{a}}\right.$ $\left[\mathrm{A}^{-}\right]=C_{A^{-}}-[\mathrm{HA}]_{2}+\left[\mathrm{A}^{-}\right]_{1} \approx C_{A^{-}}=\boldsymbol{C}_{\boldsymbol{b}}$
$\rightarrow \boldsymbol{C}_{\boldsymbol{a}}$ and $\boldsymbol{C}_{\boldsymbol{b}}$ - total concentrations of the acid and base components of the buffer

$$
\left[\mathrm{H}_{3} \mathrm{O}^{+}\right]=K_{a} \frac{[\mathrm{HA}]}{\left[\mathrm{A}^{-}\right]} \quad \Rightarrow \quad\left[\mathrm{H}_{3} \mathrm{O}^{+}\right]=K_{a} \frac{C_{a}}{C_{b}}
$$

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\(\rightarrow\) Take a \(-\log \left(0\right.\) of \(\left[\mathrm{H}_{3} \mathrm{O}^{+}\right]=K_{a}\left(C_{a} / C_{b}\right)\)
\(-\log \left[\mathrm{H}_{3} \mathrm{O}^{+}\right]=-\log K_{a}-\log \frac{C_{a}}{C_{b}}=-\log K_{a}+\log \frac{C_{b}}{C_{a}}\)
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$\Rightarrow p H=p K_{a}+\log \frac{C_{b}}{C_{a}} \quad \begin{aligned} & \text { Henderson-Hasselbalch } \\ & \text { Equation }\end{aligned}$
$\rightarrow$ The equation is valid only if $C_{a}$ and $C_{b}$ are large enough so that the assumptions [HA] $\approx C_{a}$ and [A-] $\approx C_{b}$ are justified; If not, [HA] and [A-] must be used $\rightarrow$ Works if $\boldsymbol{C}_{a}>\mathbf{1 0 0} \times \boldsymbol{K}_{a}$ and $\boldsymbol{C}_{b}>\mathbf{1 0 0} \times \boldsymbol{K}_{b}$ $\rightarrow$ If $\boldsymbol{C}_{\boldsymbol{a}}=\boldsymbol{C}_{\boldsymbol{b}}, \log \left(C_{a} / C_{b}\right)=0$ and $\boldsymbol{p} \boldsymbol{H}=\boldsymbol{p} \boldsymbol{K}_{\boldsymbol{a}}$ $\Rightarrow$ If $\boldsymbol{C}_{\boldsymbol{a}}$ and $\boldsymbol{C}_{\boldsymbol{b}}$ are comparable, the $\boldsymbol{p H}$ of the buffer is close to the $\boldsymbol{p} \boldsymbol{K}_{\boldsymbol{a}}$ of the acid component

Example: For $\mathbf{1 . 0} \mathrm{L}$ of the same buffer, calculate the $\boldsymbol{p} \boldsymbol{H}$ after the addition of $\mathbf{5 . 0} \mathbf{~ m L ~} \mathbf{2 . 0} \mathbf{M ~ H C l}$.
$\rightarrow \mathrm{HCl}$ is a strong acid and converts to $\mathrm{H}_{3} \mathrm{O}^{+}$which reacts with $\mathrm{F}^{-}$from the buffer
$\rightarrow$ Calculate the starting moles of HF and $\mathrm{F}^{-}$in the buffer and the added moles of $\mathrm{H}_{3} \mathrm{O}^{+}$from HCl
HF $\quad \rightarrow 1.0 \mathrm{~L} \times 0.50 \mathrm{~mol} / \mathrm{L}=\mathbf{0 . 5 0} \mathbf{~ m o l}$
$\mathrm{F}^{-} \quad \rightarrow 1.0 \mathrm{~L} \times 0.50 \mathrm{~mol} / \mathrm{L}=\mathbf{0 . 5 0} \mathbf{~ m o l}$
$\mathrm{H}_{3} \mathrm{O}^{+} \rightarrow 0.0050 \mathrm{~L} \times 2.0 \mathrm{~mol} / \mathrm{L}=\mathbf{0 . 0 1 0} \mathbf{~ m o l}$
$\rightarrow$ Use an "srf" table (starting, reacted, final) to calculate the final moles of HF and $\mathrm{F}^{-}$

Example: What is the $\boldsymbol{p H}$ of a buffer that is $\mathbf{0 . 5 0}$ $\mathbf{M}$ in HF and $\mathbf{0 . 5 0} \mathbf{M}$ in KF? $\left(\boldsymbol{K}_{a}=6.8 \times 10^{-4}\right.$ for HF)

1. Use the Hend.-Hass. Eq. (HF - acid; $\mathrm{F}^{-}$- base)

$$
\boldsymbol{C}_{\boldsymbol{a}}=0.50 \quad \boldsymbol{C}_{\boldsymbol{b}}=0.50
$$

$\boldsymbol{p} \boldsymbol{H}=\boldsymbol{p} \boldsymbol{K}_{\boldsymbol{a}}+\log (0.50 / 0.50)=\boldsymbol{p} \boldsymbol{K}_{\boldsymbol{a}}+\log (1)=\boldsymbol{p} \boldsymbol{K}_{\boldsymbol{a}}+0$
$\boldsymbol{p H}=\boldsymbol{p} \boldsymbol{K}_{\boldsymbol{a}}=-\log \left(6.8 \times 10^{-4}\right)=\mathbf{3 . 1 7}$
2. Or use an ice table

| [] $\mathrm{HF}+\mathrm{H}_{2} \mathrm{O} \leftrightarrow \mathrm{H}_{3} \mathrm{O}^{+}+\mathrm{F}^{-}$ |  |  |  | $K_{a}=\frac{\left[\mathrm{H}_{3} \mathbf{O}^{+}\right]\left[\mathrm{F}^{-}\right]}{[\mathrm{HF}]}=$ |
| :---: | :---: | :---: | :---: | :---: |
| $i$ | 0.50 | 0 | 0.50 |  |
| c | -x | + $x$ | + $x$ | $\underline{x(0.50+x)} \approx \frac{x(0.50)}{(0.50}$ |
| $e$ | 0.50-x | $x$ | $0.50+x$ | $(0.50-x) \quad$ (0.50) |

$\boldsymbol{x}=\boldsymbol{K}_{\boldsymbol{a}}=6.8 \times 10^{-4}=\left[\mathrm{H}_{3} \mathrm{O}^{+}\right] \Rightarrow \boldsymbol{p} \boldsymbol{H}=-\log \left(6.8 \times 10^{-4}\right)=\mathbf{3 . 1 7}$

|  | [] | $\mathrm{F}^{-}+\mathrm{H}_{3} \mathrm{O}^{+} \rightarrow \mathrm{H}_{2} \mathrm{O}+\mathrm{HF}$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| v | $s$ | 0.50 | 0.010 | - | 0.50 |
| $\stackrel{0}{0}$ | $r$ | -0.010 | -0.010 | - | +0.010 |
| : | $f$ | 0.49 | 0.00 | - | 0.51 |

$\rightarrow$ Use the Hend.-Hass. Eq. (HF - acid; F- - base)
$\rightarrow$ Since the acid an base components of the buffer are in the same volume, the ratio of the concentrations is the same as the ratio of the moles $\rightarrow \boldsymbol{C}_{\boldsymbol{b}} / \boldsymbol{C}_{\boldsymbol{a}}=\boldsymbol{n}_{\boldsymbol{b}} / \boldsymbol{n}_{\boldsymbol{a}}$

$$
\begin{aligned}
\boldsymbol{p H} & =p K_{a}+\log \frac{n_{b}}{n_{a}}=-\log \left(6.8 \times 10^{-4}\right)+\log \frac{0.49}{0.51} \\
& =3.17+(-0.02)=3.15
\end{aligned}
$$

$\Rightarrow$ The $p H$ is reduced by only 0.02 pH -units

- The addition of strong acids or bases to unbuffered solutions has a much greater effect on $\boldsymbol{p} \boldsymbol{H}$
Example: Calculate the $\boldsymbol{p H}$ after the addition of $5.0 \mathbf{~ m L ~ 2 . 0 ~ M ~ H C l ~ t o ~} 1.0 \mathrm{~L}$ of pure water .
$\rightarrow$ The $\boldsymbol{p H}$ of pure water is $\mathbf{7 . 0 0}$
$\rightarrow$ After addition of the strong acid HCl :

$$
\left[\mathrm{H}_{3} \mathrm{O}^{+}\right]=\frac{0.0050 \mathrm{~L} \times \frac{2.0 \mathrm{~mol}}{1 \mathrm{~L}}}{(1.0+0.0050) \mathrm{L}}=0.010 \mathrm{M}
$$

$$
p H=-\log (0.010)=2.00
$$

$\Rightarrow$ The $\boldsymbol{p H}$ is reduced by 5.00 pH -units (a much
larger change compared to the buffered solution)

## Preparing Buffers

$>$ Choose the conjugate acid-base pair (select a pair with an acid component having $\boldsymbol{p} \boldsymbol{K}_{\boldsymbol{a}}$ close to the desired buffer $\boldsymbol{p H}$ )
$\Rightarrow$ Calculate the ratio of $\boldsymbol{C}_{b} / \boldsymbol{C}_{\boldsymbol{a}}$ needed to achieve the desired buffer $\boldsymbol{p} \boldsymbol{H}$ (use the Hend.-Hass. Eq.)
$>$ Determine the buffer concentration and the amounts of the two components to be mixed
$>$ Mix the components and adjust the $\mathbf{p H}$ (final $p H$ may be slightly off and can be adjusted by adding strong acid or base)
$>$ Buffers can also be prepared by partial neutralization of weak acids (or bases) with strong bases (or acids) (Ex: $\mathrm{HF}+\mathrm{KOH} \rightarrow \mathrm{KF}+\mathrm{H}_{2} \mathrm{O}$; If only half of the HF is converted to $\mathrm{KF} \rightarrow$ buffer)

## Buffer Capacity and Buffer Range

- Buffer capacity ( $\boldsymbol{B C}$ ) - a measure of the ability of the buffer to resist $\boldsymbol{p H}$ changes $-\boldsymbol{B C}$ is higher for more concentrated buffers $\uparrow \uparrow C_{a}$ and $C_{b} \Rightarrow \uparrow \uparrow$ Buffer capacity
$-\boldsymbol{B C}$ is higher for buffers with similar component concentrations ( $\boldsymbol{B C}$ is highest when $\boldsymbol{C}_{\boldsymbol{a}}=\boldsymbol{C}_{b}$ )
- Buffer range ( $\boldsymbol{B R}$ ) - buffers act most
efficiently when $\boldsymbol{C}_{\boldsymbol{b}} / \boldsymbol{C}_{\boldsymbol{a}}$ is between $\mathbf{0 . 1}$ and $\mathbf{1 0}$
$\begin{aligned} & p H=p K_{a}+\log (0.1)=p K_{a}-1 \\ & p H=p K_{a}+\log (10)=p K_{a}+1\end{aligned} \Rightarrow B R=p K_{a} \pm 1$
$p H=p K_{a}+\log (10)=p K_{a}+1$

Example: How many moles of $\mathrm{NH}_{4} \mathrm{Cl}$ must be added to $\mathbf{1 . 0} \mathbf{L}$ of $\mathbf{0 . 2 0} \mathbf{M ~ N H}_{3}$ solution to get a buffer with $\boldsymbol{p} \boldsymbol{H}=\mathbf{9 . 3 5}$ ? $\left(\boldsymbol{K}_{\boldsymbol{b}}=\mathbf{1 . 8} \times \mathbf{1 0} \mathbf{0}^{-5}\right.$ for $\left.\mathrm{NH}_{3}\right)$
$\rightarrow$ Conjugate pair $\mathbf{N H}_{4}{ }^{+} / \mathbf{N H}_{3}$
$\rightarrow \boldsymbol{K}_{a}=\boldsymbol{K}_{w} / \boldsymbol{K}_{\boldsymbol{b}}=10^{-14} / 1.8 \times 10^{-5}=\mathbf{5 . 6} \times \mathbf{1 0}^{-10}$
$\rightarrow \boldsymbol{p} \boldsymbol{K}_{\boldsymbol{a}}=-\log \left(5.6 \times 10^{-10}\right)=\mathbf{9 . 2 5}$
$\rightarrow \boldsymbol{n}_{\boldsymbol{b}}=1.0 \mathrm{~L} \times 0.20 \mathrm{~mol} / \mathrm{L}=\mathbf{0 . 2 0} \mathrm{mol} \mathrm{NH}_{3} \quad \boldsymbol{n}_{a}=$ ? ?
$p H=p K_{a}+\log \frac{n_{b}}{n_{a}} \quad \rightarrow \quad 9.35=9.25+\log \frac{0.20}{n_{a}}$
$\rightarrow 0.10=\log \left(0.20 / n_{a}\right) \quad \rightarrow \quad 10^{0.10}=0.20 / n_{a}$
$\rightarrow \boldsymbol{n}_{\boldsymbol{a}}=\mathbf{0 . 2 0} / \mathbf{1 0} \mathbf{0}^{\mathbf{0 . 1 0}}=\mathbf{0 . 1 6} \mathrm{mol} \mathrm{NH}_{4}^{+}=\mathbf{0 . 1 6} \mathrm{mol} \mathrm{NH}_{4} \mathrm{Cl}$

