19.4 Equilibria Involving Complex Ions

- Complex ions consist of a metal cation surrounded by anions or neutral molecules called ligands
- Complex ions are **Lewis acid-base complexes** (adducts)
 - The metal cation is a Lewis acid
 - The ligands are Lewis bases

Formation of Complex Ions

For a metal cation, M^{n+} , and a neutral ligand, L, the complex formation is given by the equilibrium:

$$M^{n+} + mL \longleftrightarrow M(L)_m^{n+}$$

$$K_f = \frac{[\mathbf{M}(\mathbf{L})_m^{n+}]}{[\mathbf{M}^{n+}][\mathbf{L}]^m}$$

 $K_f = \frac{[\mathbf{M}(\mathbf{L})_m^{n+}]}{[\mathbf{M}^{n+}][\mathbf{L}]^m}$ $K_f \to \text{formation constant}$ of the complex

- Complexes between good Lewis acids (such as highly charged and transition metal cations) and good Lewis bases (such as NH₃, CN⁻, OH⁻, ...) are extremely stable
 - Their K_f values are very large (>10⁵)
 - ⇒If enough ligand is present in the solution, almost the entire amount of the cation is converted to the complex $\rightarrow C_M \approx [M(L)_m^{n+1}]$

Example: What is the concentration of Fe^{3+} after mixing of 25 mL 0.020 M Fe(NO₃)₃ solution with **25 mL 1.0 M** KCN solution? $(K_f = 4.0 \times 10^{43})$ for the complex Fe(CN)₆³-)

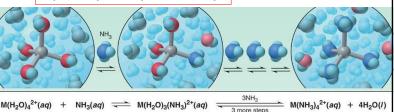
- $\rightarrow 0.020 \text{ M Fe(NO}_3)_3 \rightarrow 0.020 \text{ M Fe}^{3+}$
- $\rightarrow 1.0 \text{ M KCN} \rightarrow 1.0 \text{ M CN}^{-}$

 \triangleright In reality, the metal cation, \mathbf{M}^{n+} , is hydrated by several water molecules and the ligand, L, gradually replaces them one at a time in stepwise manner

Note: Water has been omitted from the expressions

$$\oplus$$
 $M^{n+} + mL \leftrightarrow M(L)_m^{n+}$

$$\boldsymbol{K}_f = \boldsymbol{K}_{f1} \times \boldsymbol{K}_{f2} \times ... \times \boldsymbol{K}_{fm}$$



 \rightarrow After mixing, the total volume is 25 + 25 = 50 mL

$$\Rightarrow C_{Fe} = (0.020 \text{ M} \times 25 \text{ mL/} 50 \text{ mL}) = 0.010 \text{ M}$$

$$\Rightarrow$$
 $C_{CN} = (1.0 \text{ M} \times 25 \text{ mL/}50 \text{ mL}) = 0.50 \text{ M}$

| | $Fe^{3+} + 6CN^{-} \leftrightarrow Fe(CN)_{6}^{3-}$ | | | |
|----|---|-------------|-------|---|
| i | 0.010 | 0.50 | 0 | Λ |
| c | -x | -6 <i>x</i> | +x | |
| e | 0.010 - x | 0.50 - 6x | x | K |
| e' | y | 0.44 | 0.010 | |

$$K_f = \frac{[\text{Fe}(\text{CN})_6^{3-}]}{[\text{Fe}^{3+}][\text{CN}^-]^6}$$
$$K_c = \frac{0.010}{1.000}$$

 $\rightarrow K_f$ is very large so almost the entire amount of Fe³⁺ is converted to the complex

$$\Rightarrow x \approx C_{Fe} \Rightarrow x \approx 0.010$$

$$\Rightarrow (0.50 - 6x) \approx (0.50 - 6 \times 0.010) = 0.44$$

 \rightarrow Define a new variable, $y = 0.010 - x = [Fe^{3+}]$

$$K_f = \frac{0.010}{y(0.44)^6} \implies y = \frac{0.010}{K_f(0.44)^6}$$

$$y = \frac{0.010}{4.0 \times 10^{43} (0.44)^6} = \boxed{3.4 \times 10^{-44} \text{ M} = [\text{Fe}^{3+}]}$$

Complex Ions and Solubility of Precipitates

- ➤ The solubility of some ionic solids in water can be improved by the addition of an appropriate ligand (such as NH₃, CN⁻, ...)
 - The ligand (L) forms a complex with the cation of the slightly soluble salt and the solubility equilibrium shifts toward further dissolution

$$MA(s) \leftrightarrow M^{n+} + A^{n-}$$
 $M^{n+} + mL \leftrightarrow M(L)_m^{n+}$

| | $AgCl(s) + 2NH_3(aq) \leftrightarrow Ag(NH_3)_2^+ + Cl^-$ | | | | | |
|---|---|-------------|----|----|--|--|
| i | excess | 0.10 | 0 | 0 | | |
| с | -S | -2 s | +s | +s | | |
| e | excess | 0.10 - 2s | S | S | | |

$$K = \frac{[\text{Ag(NH}_3)_2^+][\text{Cl}^-]}{[\text{NH}_3]^2} = \frac{s^2}{(0.10 - 2s)^2}$$

$$\frac{s}{0.1 - 2s} = \sqrt{K} \implies s = \frac{0.10\sqrt{K}}{1 + 2\sqrt{K}} = \frac{0.10\sqrt{3.1 \times 10^{-3}}}{1 + 2\sqrt{3.1 \times 10^{-3}}}$$
$$\implies s = 5.0 \times 10^{-3}$$

Note: The solubility of AgCl in 0.10 M NH₃ is higher than that in **pure water** which is:

$$s = (K_{sp})^{1/2} = (1.8 \times 10^{-10})^{1/2} = 1.3 \times 10^{-5} M$$

Example: The solubility of AgCl can be improved by addition of ammonia, NH₃.

$$AgCl(s) \leftrightarrow Ag^{+} + Cl^{-} \qquad \mathbf{K}_{sp} = 1.8 \times 10^{-10}$$

$$Ag^{+} + 2NH_{3}(aq) \rightarrow Ag(NH_{3})_{2}^{+} \qquad \mathbf{K}_{f} = 1.7 \times 10^{7}$$

$$\oplus$$
 AgCl(s) + 2NH₃(aq) \leftrightarrow Ag(NH₃)₂⁺ + Cl⁻ (K)

$$\rightarrow K = K_{sp} \times K_f = 1.8 \times 10^{-10} \times 1.7 \times 10^7 = \boxed{3.1 \times 10^{-3}}$$

- \rightarrow The overall equilibrium constant, K, is much larger than K_{sp}
- ⇒The addition of NH₃ shifts the equilibrium to the right and the **solubility increases**

Example: Calculate the molar solubility of AgCl in **0.10 M** NH₃ solution.

Complex Ions of Amphoteric Hydroxides

- ➤ Metals that form amphoteric oxides also form **amphoteric hydroxides** which react with both acids and bases
 - ➤ Have low solubility in pure water
 - ➤ Dissolve well in aqueous acids or bases
 - \rightarrow The solubility in acids is due to a reaction with H_3O^+ which shifts the solubility equilibrium toward dissolution (as discussed in 19.3)
 - →The solubility in bases is due to the formation of soluble complexes of the metal ions with OH-

➤ The solubility of amphoteric hydroxides can be explained as a complex formation process

Example: Al(OH)₃

- →In acidic solution OH⁻ ions are scarce so Al³⁺ is completely hydrated as $Al(H_2O)_6^{3+}$
- →As the solution becomes less acidic and more basic, the OH⁻ ions gradually accept H⁺ from the hydrating water molecules

$$\begin{split} & \text{Al}(\text{H}_2\text{O})_6^{3+} + \text{OH}^- \longleftrightarrow \text{Al}(\text{H}_2\text{O})_5\text{OH}^{2+} + \text{H}_2\text{O}(\text{I}) \\ & \text{Al}(\text{H}_2\text{O})_5\text{OH}^{2+} + \text{OH}^- \longleftrightarrow \text{Al}(\text{H}_2\text{O})_4(\text{OH})_2^+ + \text{H}_2\text{O}(\text{I}) \\ & \text{Al}(\text{H}_2\text{O})_4(\text{OH})_2^+ + \text{OH}^- \longleftrightarrow \text{Al}(\text{H}_2\text{O})_3(\text{OH})_3(\text{s}) + \text{H}_2\text{O}(\text{I}) \end{split}$$

19.5 Applications of Ionic Equilibria Selective Precipitation

- ➤ Mixtures of cations can be separated by selective precipitation with an anion if the solubilities of the precipitates are significantly different
 - The concentration of the anion is selected in a way so that Q_{sp} is above K_{sp} for the less soluble ion and just below K_{sp} for the more soluble ion

Example: What is the maximum concentration of Cl⁻ that would precipitate only one of the ions in a solution that is **0.0010 M** in Ag⁺ and **0.020 M** in Pb²⁺? $K_{sp}(AgCl) = 1.8 \times 10^{-10}$; $K_{sp}(PbCl_2) = 1.7 \times 10^{-5}$

- →The neutral complex of **Al** is insoluble
 - \Rightarrow A precipitate forms \rightarrow Al(H₂O)₃(OH)₃(s) \rightarrow usually written without the water as Al(OH)₃(s)
- →As the solution becomes even more basic, the OHions accept H⁺ from one of the 3 remaining water molecules

$$Al(H_2O)_3(OH)_3(s) + OH^- \leftrightarrow Al(H_2O)_2(OH)_4^- + H_2O(l)$$

- ⇒The precipitate dissolves due to the formation of the soluble negative complex ion
- $ightharpoonup Zn^{2+}$, Sn^{2+} , Pb^{2+} , ... exhibit similar behavior
- ➤ Other metals such as Fe²⁺, Fe³⁺, ... only form the neutral insoluble hydroxide complex which can not be dissolved in basic solution

$$AgCl(s) \rightarrow Ag^{+} + Cl^{-}$$
 $K_{sp} = [Ag^{+}][Cl^{-}]$
 $PbCl_{2}(s) \rightarrow Pb^{2+} + 2Cl^{-}$ $K_{sp} = [Pb^{2+}][Cl^{-}]^{2}$

→ Calculate the concentrations of Cl⁻ at which precipitation of each ion begins:

$$[Cl^{-}] = \frac{K_{sp}}{[Ag^{+}]} = \frac{1.8 \times 10^{-10}}{0.0010} = 1.8 \times 10^{-7} \text{ M}$$

$$[CI^{-}] = \sqrt{\frac{K_{sp}}{[Pb^{2+}]}} = \sqrt{\frac{1.7 \times 10^{-5}}{0.020}} = 2.9 \times 10^{-2} \text{ M}$$

- \rightarrow AgCl will precipitate first at [Cl⁻] > 1.8×10⁻⁷ M
- \rightarrow If [Cl⁻] < 2.9×10⁻² M, PbCl₂ will not precipitate

 \Rightarrow The maximum [Cl⁻] is just below $\boxed{2.9 \times 10^{-2} \text{ M}}$

Note: The concentration of unprecipitated Ag⁺ is:

$$[Ag^+] = K_{sp}/[Cl^-] = 1.8 \times 10^{-10}/2.9 \times 10^{-2} = 6.2 \times 10^{-9} \text{ M}$$